Question 8 (6 marks)

A particle moves in space with position vector cm, where is the time in seconds since its motion began.

(a) Determine the distance of the particle from its initial position after seconds. (3 marks)

(b) Show that the particle is moving with a constant speed. (3 marks)

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|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ position vectors at and required time  ü displacement vector  ü correct distance |

(b) Show that the particle is moving with a constant speed. (3 marks)

|  |
| --- |
| Solution |
| Velocity vector:  Speed:  Hence particle is moving with a constant speed. |
| Specific behaviours |
| ü correct velocity vector  ü correct expression for magnitude of vector  ü simplifies magnitude to show constant |

Question 12 (7 marks)

Relative to an origin located on level ground, a projectile is launched from m with an initial velocity of m/s. The motion of the projectile is only affected by a constant acceleration of m/s2.

(a) Derive from the acceleration vector an expression for the position vector of the projectile s after its launch. (3 marks)

(b) Determine the distance travelled through the air by the projectile from when it is launched until the instant it reaches the ground, correct to the nearest m. (4 marks)

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(a) Derive from the acceleration vector an expression for the position vector of the projectile s after its launch. (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ antidifferentiates acceleration, shows constant  ü antidifferentiates velocity, shows constant  ü correctly uses initial conditions to evaluate constants |

(b) Determine the distance travelled through the air by the projectile from when it is launched until the instant it reaches the ground, correct to the nearest m. (4 marks)

|  |
| --- |
| Solution |
| Reaches ground level when vertical component of position is :  Distance travelled: |
| Specific behaviours |
| ✓ equation for time to reach ground level  ü obtains time to reach ground level  ü integral for distance travelled  ü correct distance travelled |

Question 13 (7 marks)

At time seconds, , the position vector m of a particle is given by

(a) State the position vector of the point that the particle approaches as . (1 mark)

(b) Determine the speed of the particle when , correct to the nearest m/s.

(3 marks)

(c) Express the Cartesian equation for the path of the particle in the form . (3 marks)

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|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct position vector |

(b) Determine the speed of the particle when , correct to the nearest m/s.

(3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ obtains velocity vector  ü velocity vector at required time  ü calculates magnitude of velocity |

(c) Express the Cartesian equation for the path of the particle in the form . (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ obtains expression for in terms of  ü obtains , simplification optional  ü uses initial position and part (a) to state domain restriction |

Question 15 (10 marks)

An aeroplane flying at a constant altitude releases a bomb at with an initial velocity of The path of the bomb is shown below.

<EFOFEX>
id:fxd{ed477769-0541-4ed0-b389-071f37b300f3}

FXData:

</EFOFEX>

Assume there is no wind in the region, air resistance can be ignored and the only acceleration acting on the bomb is ms-2 due to gravity.

(a) Use the acceleration vector of the bomb to clearly deduce that its position vector at  
time seconds after release is . (3 marks)

(b) Determine the speed of the bomb seconds after it is released. (2 marks)

Five seconds after the bomb is released, a projectile is launched from the origin with a speed of at an angle of elevation of to intercept it at a height of m.

The position vector of the projectile seconds after its launch is

(c) Determine the value of and the value of so that the projectile intercepts the bomb.

(5 marks)

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(a) Use the acceleration vector of the bomb to clearly deduce that its position vector at  
time seconds after release is . (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ü correct integration of acceleration vector, with constant  ✓ clearly shows use of initial conditions to obtain velocity vector  ü repeats with velocity vector to obtain position vector |

(b) Determine the speed of the bomb seconds after it is released. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates velocity  ü calculates correct speed |

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The position vector of the projectile seconds after its launch is

(c) Determine the value of and the value of so that the projectile intercepts the bomb.

(5 marks)

|  |
| --- |
| Solution |
| Bomb reaches m when  Horizontal position of bomb is m.  Projectile will travel for seconds.  Horizontal position of projectile  Vertical position of projectile  Hence  And |
| Specific behaviours |
| ✓ calculates time of interception  ü uses horizontal position of interception to form equation  ü uses vertical position of interception to form equation  ü solves for angle in degrees or radians  ü solves for initial speed |

Question 18 (8 marks)

<EFOFEX>
id:fxd{31127fe7-e423-4d5d-a422-11758e61f25b}

FXData:
</EFOFEX>The path of a small submersible moving  
below the surface of the sea (the -axis)  
is shown in the diagram, where is the  
time in seconds and .  
The position vector of the submersible is

(a) State, with reasoning, whether the submersible is moving from left to right or from right to left. (2 marks)

(b) Determine the Cartesian equation for the path of the submersible. (3 marks)

(c) Determine the distance travelled by the submersible when its depth below the surface is at least metres, correct to the nearest centimetre. (3 marks)

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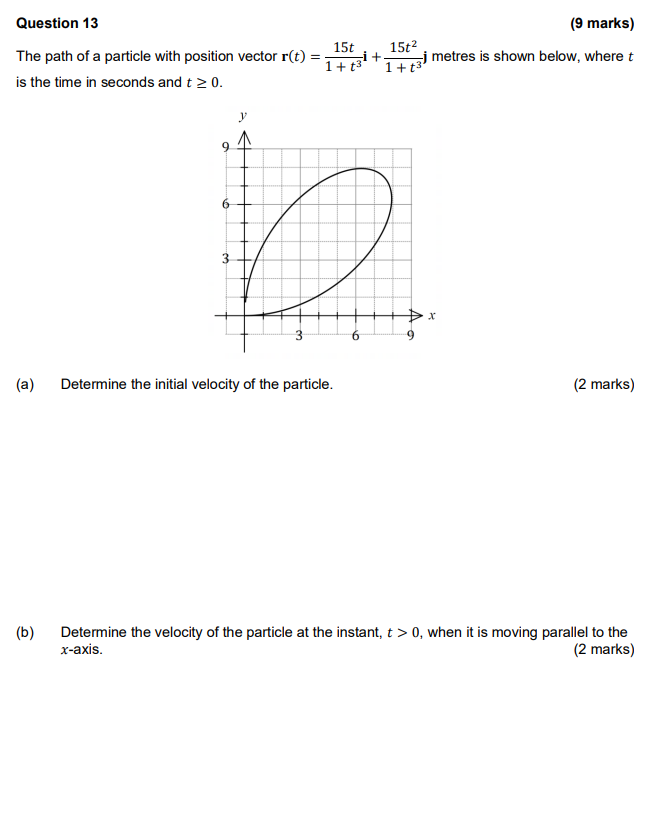
|  |
| --- |
| Solution |
| The -coefficient will always be negative and so submersible is moving from right to left. |
| Specific behaviours |
| ✓ differentiates to obtain velocity vector  ü states right to left, with reason |

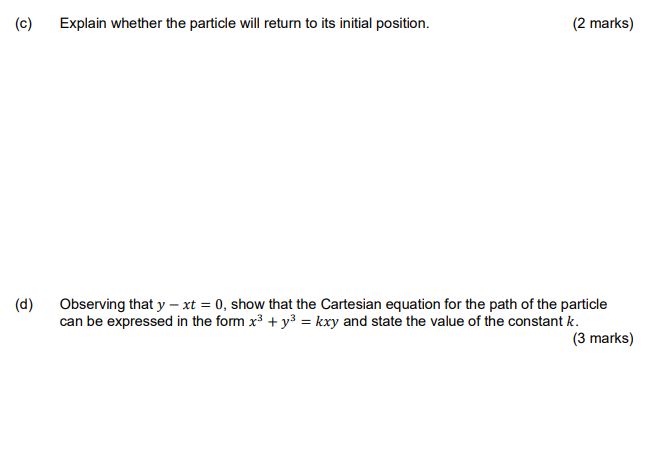
(b) Determine the Cartesian equation for the path of the submersible. (3 marks)

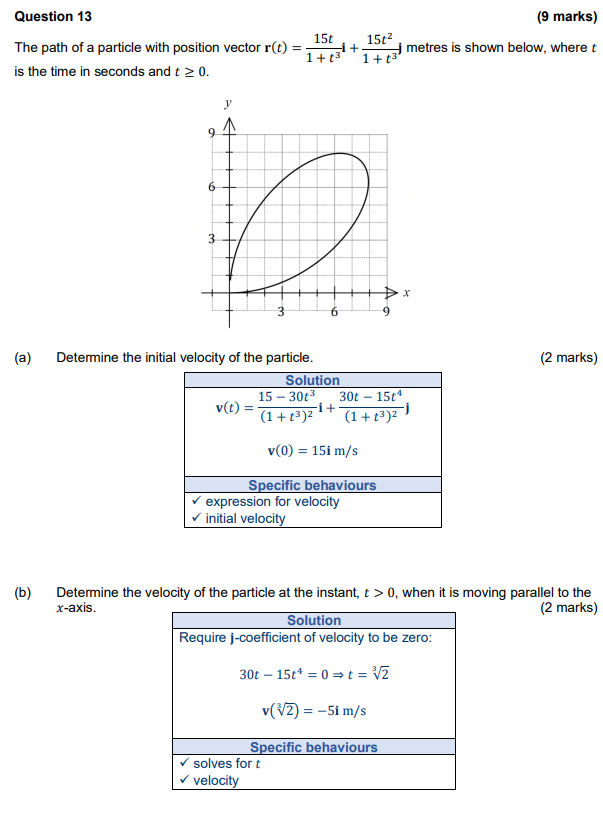
|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ obtains suitable trigonometric identity  ü sets , and arranges equations for use with identity  ü eliminates trigonometric terms and simplifies |

(c) Determine the distance travelled by the submersible when its depth below the surface is at least metres, correct to the nearest centimetre. (3 marks)

|  |
| --- |
| Solution |
| Depth is at least m when  Distance\*:  *\* Will take seconds to evaluate using numerical integration* |
| Specific behaviours |
| ✓ obtains correct time interval  ü writes integral using magnitude of velocity  ü obtains correct distance, with units |

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